AP Calculus Summer Packet
Mrs. Russell's Class

Please complete the following problems BEFORE your first day of AB Calculus . If you have any questions, contact Mrs. Russell at amanda.russell@maryville-schools.org

Formula Sheet

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$$

$$\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim\limits_{h \to \infty} \dfrac{f(x+h) - f(x)}{h}$

Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: Ax + By + C = 0

I. Complex Fractions

When simplifying complex fractions, multiply by a fraction equal to 1 which has a numerator and denominator composed of the common denominator of all the denominators in the complex fraction.

Worked out examplex:

$$\frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} = \frac{-7-\frac{6}{x+1}}{\frac{5}{x+1}} \cdot \frac{x+1}{x+1} = \frac{-7x-7-6}{5} = \frac{-7x-13}{5}$$

$$\frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} = \frac{\frac{-2}{x}+\frac{3x}{x-4}}{5-\frac{1}{x-4}} \cdot \frac{x(x-4)}{x(x-4)} = \frac{-2(x-4)+3x(x)}{5(x)(x-4)-1(x)} = \frac{-2x+8+3x^2}{5x^2-20x-x} = \frac{3x^2-2x+8}{5x^2-21x}$$

Simplify each of the following.

1. $\dfrac{\frac{25}{a}-a}{5+a}$

2. $\dfrac{4-\frac{12}{2x-3}}{5+\frac{15}{2x-3}}$

3. $\dfrac{\frac{x}{x+1}-\frac{1}{x}}{\frac{x}{x+1}+\frac{1}{x}}$

II. Functions

To evaluate a function for a given value, simply plug the value into the function for x.

$$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2+1 \\ &= 2(x^2-8x+16)+1 \\ &= 2x^2-16x+32+1 \\ f(g(x)) &= 2x^2-16x+33 \end{aligned}$$

Let $f(x)=2x+1$ *and* $g(x)=2x^2-1$. **Find each.**

4. $g(-3) =$ ______________ 5. $f(t+1) =$ __________ 6. $f\left[g(-2)\right] =$ __________

7. $g\left[f(m+2)\right] =$ ___________ 8. $\dfrac{f(x+h)-f(x)}{h} =$ ____

Find $\dfrac{f(x+h)-f(x)}{h}$ **for the given function *f*.**

9. $f(x) = 9x + 3$

10. $f(x) = 5 - 2x^2$

III. Intercepts and Points of Intersection

To find the x-intercepts, let y = 0 in your equation and solve.
To find the y-intercepts, let x = 0 in your equation and solve.

Example: $y = x^2 - 2x - 3$

$\underline{x-\text{int}.\ (Let\ y=0)}$

$0 = x^2 - 2x - 3$

$0 = (x-3)(x+1)$

$x = -1\ or\ x = 3$

$x-\text{intercepts}\ (-1,0)\ and\ (3,0)$

$\underline{y-\text{int}.\ (Let\ x=0)}$

$y = 0^2 - 2(0) - 3$

$y = -3$

$y-\text{intercept}\ (0,-3)$

Find the x and y intercepts for each.

11. $y = x^2 + x - 2$

12. $y^2 = x^3 - 4x$

Find the point(s) of intersection of the graphs for the given equations. (hint: use substitution and/or elimination)

13. $$\begin{aligned} x^2 + y &= 6 \\ x + y &= 4 \end{aligned}$$

14. $$\begin{aligned} x^2 - 4y^2 - 20x - 64y - 172 &= 0 \\ 16x^2 + 4y^2 - 320x + 64y + 1600 &= 0 \end{aligned}$$

IV. Interval Notation

15. Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2 < x \leq 4$		
	$[-1, 7)$	
		

V. Domain and Range

Find the domain and range of each function. Write your answer in INTERVAL notation.

16. $f(x) = x^2 - 5$

17. $f(x) = -\sqrt{x+3}$

18. $f(x) = 3\sin x$

19. $f(x) = \dfrac{2}{x-1}$

VI. **<u>Inverses</u>**

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.
Example:

$f(x)=\sqrt[3]{x+1}$	Rewrite f(x) as y
$y=\sqrt[3]{x+1}$	Switch x and y
$x=\sqrt[3]{y+1}$	Solve for your new y
$(x)^3=\left(\sqrt[3]{y+1}\right)^3$	Cube both sides
$x^3=y+1$	Simplify
$y=x^3-1$	Solve for y
$f^{-1}(x)=x^3-1$	Rewrite in inverse notation

Find the inverse for each function.

20. $f(x)=2x+1$

21. $f(x)=\dfrac{x^2}{3}$

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
$f(g(x))=g(f(x))=x$

Prove *f and g* are inverses of each other.

22. $f(x)=\dfrac{x^3}{2}$ $\quad g(x)=\sqrt[3]{2x}$

23. $f(x)=9-x^2,\ x\ge 0$ $\quad g(x)=\sqrt{9-x}$

VII. Equation of a line

Slope intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Vertical line: x = c (slope is undefined)

Horizontal line: y = c (slope is 0)

24. Determine the equation of a line passing through the point (5, -3) with an undefined slope.

25. Determine the equation of a line passing through the point (-4, 2) with a slope of 0.

26. Find the equation of a line passing through the point (2, 8) and parallel to the line $y = \frac{5}{6}x - 1$.

27. Find the equation of a line perpendicular to the y- axis passing through the point (4, 7).

28. Find the equation of a line with an x-intercept (2, 0) and a y-intercept (0, 3).

VIII. Trigonometry
Radian and Degree Measure

Use $\frac{180^\circ}{\pi\, radians}$ to get rid of radians and convert to degrees.

Use $\frac{\pi\, radians}{180^\circ}$ to get rid of degrees and convert to radians.

29. Convert to degrees: a. $\frac{5\pi}{6}$ b. $\frac{4\pi}{5}$ c. 2.63 radians

30. Convert to radians: a. $45°$ b. $-17°$ c. 237°

Angles in Standard Position

31. Sketch the angle in standard position.

a. $\frac{11\pi}{6}$ b. $230°$ c. $-\frac{5\pi}{3}$ d. 1.8 radians

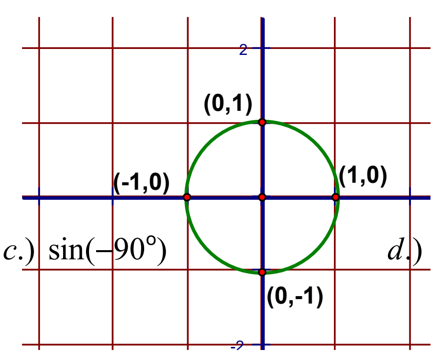
Reference Triangles

32. Sketch the angle in standard position. Draw the reference triangle and label the sides, if possible.

a. $\frac{2}{3}\pi$ b. 225° c. $-\frac{\pi}{4}$ d. 30°

Unit Circle

You can determine the sine or cosine of a quadrantal angle by using the unit circle. The x-coordinate of the circle is the cosine and the y-coordinate is the sine of the angle.

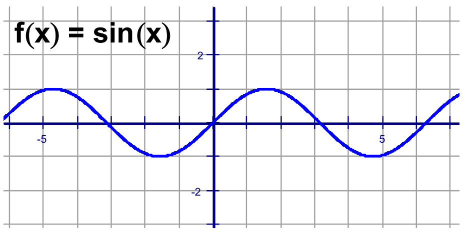


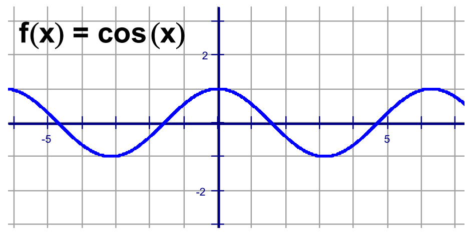
Example: $\sin 90° = 1$ $\cos\frac{\pi}{2} = 0$

33. *a.*) $\sin 180°$ *b.*) $\cos 270°$ *c.*) $\sin(-90°)$ *d.*) $\sin\pi$

$e.)\ \cos 360°$ $\qquad\qquad$ $f.)\ \cos(-\pi)$

Graphing Trig Functions





y = sin x and y = cos x have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the functions below. For $f(x) = A\sin(Bx + C) + K$, A = amplitude, $\frac{2\pi}{B}$ = period, $\frac{C}{B}$ = phase shift (positive C/B shift left, negative C/B shift right) and K = vertical shift.

Graph two complete periods of the function.

34. $f(x) = 5\sin x$

35. $f(x) = \sin 2x$

36. $f(x) = -\cos\left(x - \frac{\pi}{4}\right)$

37. $f(x) = \cos x - 3$

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

38. $\sin x = -\frac{1}{2}$

39. $2\cos x = \sqrt{3}$

40. $\cos 2x = \frac{1}{\sqrt{2}}$

41. $\sin^2 x = \frac{1}{2}$

42. $\sin 2x = -\frac{\sqrt{3}}{2}$

43. $2\cos^2 x - 1 - \cos x = 0$

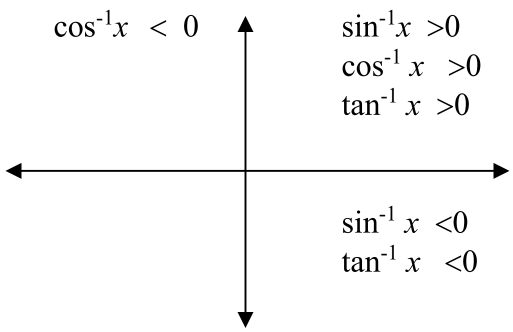
44. $4\cos^2 x - 3 = 0$

45. $\sin^2 x + \cos 2x - \cos x = 0$

Inverse Trigonometric Functions:

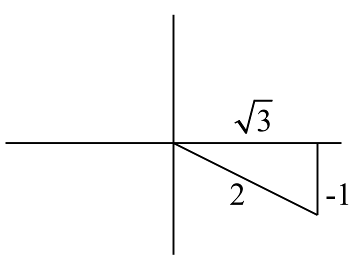
Recall: Inverse Trig Functions can be written in one of ways: $\arcsin(x)$ $\sin^{-1}(x)$

Inverse trig functions are defined only in the quadrants as indicated below due to their restricted domains.



Example:
Express the value of "y" in radians.

$y = \arctan \frac{-1}{\sqrt{3}}$ Draw a reference triangle.



This means the reference angle is 30° or $\frac{\pi}{6}$. So, y = $-\frac{\pi}{6}$ so that it falls in the interval from

$\frac{-\pi}{2} < y < \frac{\pi}{2}$ Answer: y = $-\frac{\pi}{6}$

For each of the following, express the value for "y" in radians.

46. $y = \arcsin\frac{-\sqrt{3}}{2}$

47. $y = \arccos(-1)$

48. $y = \arctan(-1)$

Example: Find the value without a calculator.

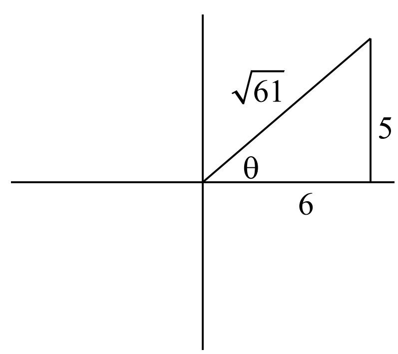
$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.

Find the missing side using Pythagorean Thm.

Find the ratio of the cosine of the reference triangle.

$$\cos\theta = \frac{6}{\sqrt{61}}$$



For each of the following give the value without a calculator.

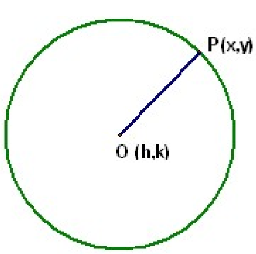
49. $\tan\left(\arccos\frac{2}{3}\right)$

50. $\sec\left(\sin^{-1}\frac{12}{13}\right)$

51. $\sin\left(\arctan\frac{12}{5}\right)$

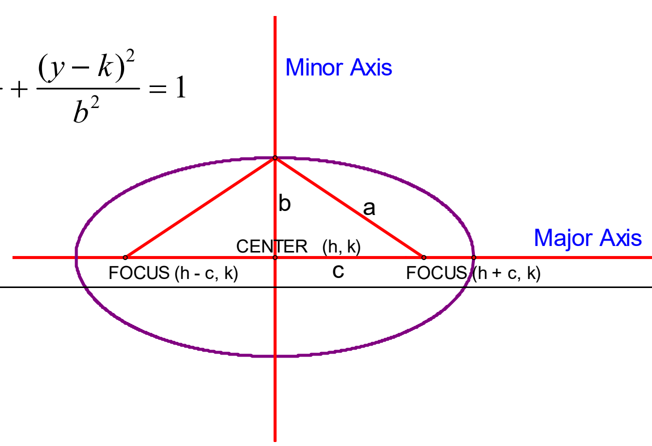
52. $\sin\left(\sin^{-1}\frac{7}{8}\right)$

IX. Circles and Ellipses



$$r^2 = (x-h)^2 + (y-k)^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

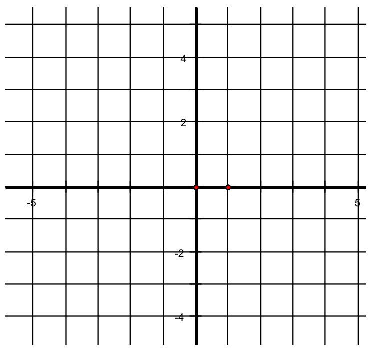


For a circle centered at the origin, the equation is $x^2 + y^2 = r^2$, where **r** is the radius of the circle.

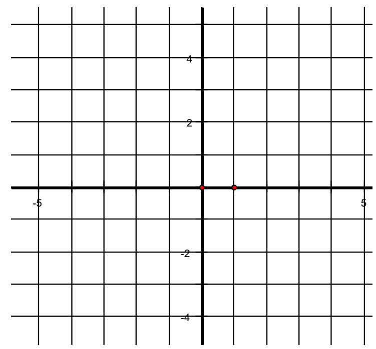
For an ellipse centered at the origin, the equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where **a** is the distance from the center to the ellipse along the x-axis and **b** is the distance from the center to the ellipse along the y-axis. If the larger number is under the y^2 term, the ellipse is elongated along the y-axis. For our purposes in Calculus, you will not need to locate the foci.

Graph the circles and ellipses below:

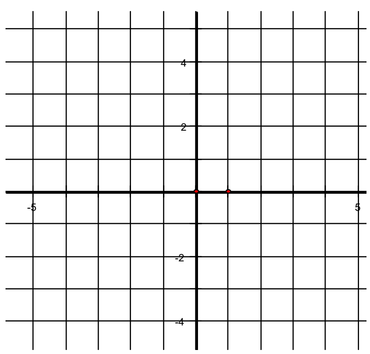
67. $x^2 + y^2 = 16$



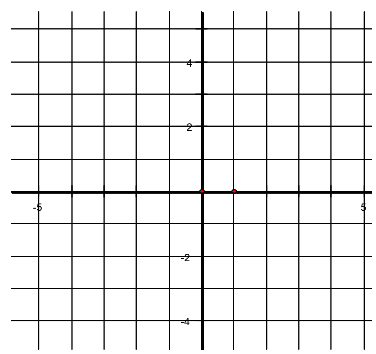
68. $x^2 + y^2 = 5$



69. $\frac{x^2}{1} + \frac{y^2}{9} = 1$



70. $\frac{x^2}{16} + \frac{y^2}{4} = 1$



X. Limits

Please watch the video below to preview limits for Calculus:

https://www.youtube.com/watch?v=ic98OGm-v3I